B.SC. SECOND YEAR, III-SEMESTER SYLLABUS

BS-303-T: PAPER-III (Theory): STATISTICAL INFERENCE

(4 HPW:: 4 Credits :: 100 Marks: (External:80 & Internal:20)

Course Objectives:

1. Able to evaluate and familiar to estimate the parameter (if exists) for the any standard probability distribution(s) using methods of maximum likelihood and method of moments, and estimation of parameters using method of least squares.
2. Able to examine the estimator satisfies the criteria of good estimation (Consistency, unbiasedness, sufficiency) for standard distributions.
3. Able to evaluate and familiar the confidence interval estimates for the parameter using pivot method.
4. Framing statistical hypothesis and knowing the statistical test procedure.
5. Identifying the suitable large / small sample parametric / non parametric tests among the basic statistical tests.

UNIT –I

Point and Interval Estimation: Review on Sampling distributions; exponential family of distributions Point estimation of a parameter, concept of bias and mean square error of an estimate. Criteria of a good estimator. Consistency (Definition, necessary conditions, problems); Unbiasedness (Definition, Median modal unbiasedness necessary conditions, problems);

UNIT –II

Sufficiency: Definition, properties, Statement of Neyman's Factorization theorem, one parameter exponential family, computation of sufficient statistics in case of Binomial, Poisson, Normal and Exponential (one parameter only) distributions. Methods of Estimation: Method of Moments and Method of Maximum likelihood estimation (MLE); their properties and computation for standard probability distributions (if exists), statements of asymptotic properties of MLE. Method of least squares, Concept of interval estimation. Confidence intervals of the parameters of normal population by Pivot method.

UNIT –III

Testing of Hypothesis: Concepts of statistical hypotheses, Statement and Proof of Neyman-Pearson's fundamental lemma for Randomized tests. Examples in case of Binomial, Poisson, Exponential and Normal distributions and their power of the test functions. **Large sample tests:** Tests for single sample mean, difference of means, single sample proportion, difference of proportions and difference of standard deviations. **O**rder statistics: Definition, statements of their distributions and applications.

UNIT – IV

Small sample Tests: Tests for single sample mean, variance, difference of means (independent and related samples), equality of variances, Test for goodness of fit and independence of attributes.

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Non-parametric tests: Advantages and disadvantages, comparison with parametric tests. Use of Central Limit Theorem for normal approximations. Single sample Sign test and Runs test, Wilcoxon-signed rank test (single and paired samples). Two independent sample tests: Median test, Wilcoxon –Mann-Whitney U test, Wald Wolfowitz's runs test**.**

Course Outcome:

1. Familiar to estimate the parameter (if exists) for the any standard probability distribution(s) using methods of maximum likelihood and method of moments, and estimation of parameters using method of least squares.
2. Examine the estimator satisfies the criteria of good estimation (Consistency, unbiasedness, sufficiency) for standard distributions.
3. Evaluate and familiar the confidence interval estimates for the parameter using pivot method.
4. Framing statistical hypothesis and knowing the statistical test procedure.
5. Identifying the suitable large/small sample parametric/non-parametric tests among the basic statistical tests.

List of Reference Books:

1. V.K. Kapoor and S.C.Gupta: Fundamentals of Mathematical Statistics, Sultan Chand & Sons,
2. Goon AM, Gupta MK,Das Gupta B : Outlines of Statistics , Vol-II, the World Press Pvt. Ltd., .
3. V.K.Rohatgi: An introduction to probability and statistics.Wiley series.
4. Sanjay Arora and BansiLal:.New Mathematical Statistics Satya Prakashan , New Delhi
5. Siegal,S.,and Sidney: Non-parametric statistics for Behavioral Science. McGraw Hill.
6. Hogg, Tanis, Rao. Probability and Statistical Inference.7th edition. Pearson Publication.

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UNIT – I

1. Define Consistency. With suitable examples.
2. State and prove sufficient conditions for consistency.
3. Show that in sampling from Normal distribution N(μ , σ^2) sample mean is a consistent estimator of μ.
4. Show that in sampling from Normal distribution N(μ , σ^2) sample variance is a consistent estimator of σ^2.
5. Show that in sampling from Normal distribution N(μ , σ^2) sample mean square is a consistent estimator of σ^2.
6. Show that in sampling from Normal distribution N(μ , σ^2) sample median is a consistent estimator of μ.
7. If $x_1, x_2, \ldots\ldots, x_n$ are random observations on a Bernoulli variate X taking the value 1 with probability P and the value 0 with probabilityP(1-P) show that $\frac{\sum xi}{n}(1-\frac{\sum xi}{n})$ is a consistent estimator of P(1-P).
8. If $x_1, x_2, \ldots\ldots, x_n$ is a random sample from normal population N(μ , 1) show that t = $\frac{1}{n}\sum_{i=1}^{n} xi^2$ is an unbiased estimator of $\mu^2 + 1$
9. Define Unbiasedness with suitable examples.
10. Show that In sampling from Normal distribution N(μ , σ^2) sample mean is an unbiased estimator of μ.
11. Show that In sampling from Normal distribution N(μ , σ^2) sample median is an unbiased estimator of μ.
12. Show that In sampling from Normal distribution N(μ , σ^2) sample variance is a biased estimator of σ^2 but sample mean square is an unbiased estimator of σ^2.
13. Define mean square error of an estimate.
14. Define point estimation of a parameter and give two examples.
15. Define bias and give two examples.
16. If T is an unbiased estimator for θ show that T^2 is a biased estimator for θ^2

UNIT - II

1. Explain method of Maximum Likelihood Estimation (MLE)?
2. Explain properties of Maximum Likelihood Estimation (MLE)?
3. In random sampling from normal population N(μ , σ^2) Obtain Maximum Likelihood estimator of μ when σ^2 is known.
4. In random sampling from normal population N(μ , σ^2) Obtain Maximum Likelihood estimator of σ^2 when μ is known.
5. Obtain Maximum Likelihood estimator of θ for an Exponential Distribution.
6. Obtain Maximum Likelihood estimator of **P** for a Binomial Distribution.
7. Obtain Maximum Likelihood estimator of λ for the Poisson Distribution
8. Explain method of Moments.
9. Obtain an estimator of " λ " and " σ^2 " for a Normal Distribution by using method of Moments.
10. Obtain an estimator of " **P** " for a Binomial Distribution by using method of Moments.
11. Obtain an estimator of " θ " for an Exponential Distribution by using method of Moments.
12. Obtain an estimator of " λ " for a Poisson distribution by using methods of Moments.
13. Define Sufficiency. State Neyman's factorization theorem.
14. Obtain Sufficient estimator of " μ " and " σ^2 " for a Normal Distribution.
15. Obtain sufficient estimator of " θ " for an Exponential Distribution.
16. Obtain sufficient estimator of " **P** " for a Binomial Distribution.
17. Obtain sufficient estimator of " λ " for a poison Distribution.
18. Define interval estimation.
19. Obtain 99% confidence interval for the population parameter " μ " in the Normal population when " σ^2 " is known.
20. Construct the confidence limits at (1 - α) 100% level using pivot method for the parameter " μ " and " σ^2 " .

UNIT - III

1. Define (i) Null Hypothesis (ii) Alternative Hypothesis (iii) Critical region (iv) Level of Significance (v) Power of the test.
2. Define Type – I and Type – II errors. Give two examples of each.
3. Define most powerful test.
4. State and prove Neyman - Pearson Lemma. Explain its importance in the testing of hypothesis.
5. Find the most powerful critical region for testing: $H_0: P = P_0$ against $H_1: P = P_1 (\neq P_0)$ for binomial distribution with parameters n and P.
6. Find the most powerful critical region for testing $H_0: \lambda = \lambda_0$ against $H_1: \lambda = \lambda_1 (\neq \lambda_0)$ for passion distribution with parameter λ.
7. Find the most powerful critical region for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (\neq \theta_0)$ for exponential distribution with parameter θ.
8. Find the most powerful critical region for testing $H_0: \mu = \mu_0$ against $H_1: \mu = \mu_1 (\neq \mu_0)$ for Normal distribution $N(\mu, \sigma^2)$ when σ^2 is known .
9. Find the most powerful critical region for testing $H_0: \sigma^2 = \sigma^2_0$ against $H_1: \sigma^2 = \sigma^2_1 (\neq \sigma^2_0)$ for Normal distribution $N(\mu, \sigma^2)$ when μ is known.
10. Write the test procedure for testing any statistical hypothesis.
11. Explain the large sample test procedure for testing a single proportion.
12. Go through the problems on single proportion.
13. Explain the large sample test procedure for testing a difference of proportions.
14. Learn problems on difference of proportions.
15. Explain the large sample test procedure for testing a single mean.
16. Learn problems on Single mean.
17. Explain the large sample test procedure for difference of means.
18. Learn problems on difference of means.
19. Explain the large sample test procedure for difference of Standard Deviations.
20. Learn problems on difference of Standard Deviations.
21. Define order statistics obtain its minimum and maximum order statistics distribution.
22. Derive the probability density function of minimum and maximum order statistic distribution.
23. Write down the probability density function formula for $\mathbf{r^{th}}$ order statistics.
24. Write any four applications of order statistics used in statistical theory.

UNIT – IV

1. Explain procedure of test statistic for chi-square test for independence of attributes.
2. Stating the conditions to apply chi-square and explain procedure of test statistic for chi-square test for goodness of fit.
3. For a 2 x 2 contingency table, prove that

$$X^2 = \frac{N(ad - bc)2}{(a+b)(a+c)(b+d)(c+d)}$$

4. Explain yate's correction for continuity in chi-square test for independence of attributes.
5. Learn problems on chi-square test for goodness of fit.
6. Learn problems on chi-square test for independence of attributes.
7. Describe F – test for testing the equality of population variances.
8. Learn problems on F – test.
9. Describe t – test for testing the equality of population means when samples are independent.
10. Learn problems on t – test for difference of means.
11. Describe t – test for testing the equality of population means when samples are dependent (paired t- test).
12. Learn problems on paired t – test.
13. Describe t – test for testing single mean.
14. Learn problems on t – test for single mean.
15. Explain the procedure of Mann - Whitney U test for two independent samples.
16. Explain the procedure of Wald Wolfowitz Run test for testing the equality of two population distributions.
17. Write the use of central limit theorem in testing.
18. Differentiate between parametric and non-parametric tests.
19. Write down advantages and disadvantages of non-parametric test.
20. Define Run and length of the Run.
21. Write the test procedure for randomness.
22. Write the test procedure for sign test.